Engineering Notes

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Building a Semi-Mechanistic Interior Ballistic Model by Experimental Design

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Nomenclature

A = bore cross sectional area = 0.03762 in.^{2}
C = calorific value of propellant = 387.27k cal/lb
E = specific energy of propellant gas
K_1, K_2, K_3 = burning rate parameters
$\hat{K}_1 \hat{K}_2 \hat{K}_3$ = estimated values of burning rate parameters
m = projectile mass = 0.00787 lb
p = pressure
p_a = pressure ahead of the projectile = 1000 psi
p_b = projectile base pressure
$p_{\min}$ = minimum starting pressure = 2530 psi
q = coefficient of heat transfer
R = displacement from the breech end
R_0 = gas constant
r = instantaneous radius of propellant particles
r_0 = initial radius of propellant particles
T = temperature
t = time
u = velocity
u_b = projectile velocity
v = gaseous specific volume
x = Lagrangian coordinate
X_1 = charge weight, in grains
X_2 = particle diameter, in inches
X_3 = initial temperature of charge, in °F
γ = ratio of specific heats = 1.25
η = specific covolume of propellant gas = 32.2 in.^{3}/lb
ρ_0 = initial mass of propellant and air per unit chamber volume, where chamber volume = 0.1088 in.^{3}
ρ_1 = specific gravity of propellant = 0.0559 lb/in.^{3}
ρ_g, ρ_p = partial density of gas and propellant, respectively

Introduction

In a previous paper,[1] a semimechanistic interior ballistic model was built for a 5.56 mm weapon using nitrocellulose charge. One-dimensional Lagrangian fluid dynamic theory was employed and the model was iteratively improved, leading to an adequate prediction of the chamber pressure-time curves observed under a single experimental condition (19 grains charge weight, 0.0263 in. propellant particle diameter and 70°F initial temperature of charge). The model was applicable only at the experimental condition considered and required modifications in view of the predicted 36% unburnt propellant, 9% higher muzzle velocity and 8000°F temperature after primer explosion. This Note presents further model improvements and extends the model to be valid over preselected ranges of charge weight, particle diameter and initial temperature of charge.

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‡The quantities $E, p, R, r, T, u, v, \rho_g, \rho_p$ are functions of t and x. p_b, u_b are functions of t alone. Others are constants.

Interior Ballistic Model

The specific model improvements are[2]: 1) incorporation of energy losses due to heat transfer by using the heat transfer equation of Nordheim et al.[3] with the coefficient of heat transfer chosen to make the total heat transfer about 30% of the projectile kinetic energy at muzzle[4]; 2) approximation of the resistance to projectile motion due to engraving, barrel friction and air compression by a minimum starting pressure followed by a constant pressure ahead of the projectile; 3) modification of the mass burning rate equation to include the dynamic form function of the charge under gun conditions; 4) elimination of ignition delay from the observed pressure-time curves; and 5) refinement of gas properties at $t=0$ by an experimental determination of primer characteristics. These characteristics are: mass = 0.00004415 lb, calorific value = 13.6k cal/lb, specific gravity = 0.072 lb/in.^{3}, covolume = 32.2 in.^{3}/lb and the ratio of specific heats = 1.25. With these modifications, the one dimensional interior ballistic model is

$$\frac{\partial R}{\partial t} = u \qquad \text{(Velocity)} \qquad (1a)$$

$$(\rho_p + \rho_g)\frac{\partial R}{\partial x} = \rho_0 \qquad \text{(Conservation of Mass)} \qquad (1b)$$

$$v = (\rho_1 - \rho_p)/\rho_1\rho_g \qquad \text{(Specific Volume)} \qquad (1c)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \qquad \text{(Conservation of Momentum)} \qquad (1d)$$

$$\frac{\partial}{\partial t}\left[\frac{\partial R}{\partial x}(E\rho_g + C\rho_p)\right] = -p\frac{\partial}{\partial t}\left(\frac{\rho_0}{\rho_p + \rho_g}\right)$$

$$-q\frac{p(v-\eta)}{R_0 v}\cdot \pi d u \cdot \frac{\partial R}{\partial x} \qquad \text{(Conservation of Energy)} \qquad (1e)$$

$$E = p(v-\eta)/(\gamma - 1) \qquad \text{(State)} \qquad (1f)$$

$$\frac{\partial}{\partial t}\left(\rho_p \frac{\partial R}{\partial x}\right) = \frac{3\rho_1}{r_0}$$

$$\times \left(\frac{\rho_p}{\rho_0}\frac{\partial R}{\partial x}\right)^{K_3} K_1 p^{K_2} \qquad \text{(Mass Burning Rate)} \qquad (1g)$$

Initial Conditions: At $t=0$, for all x; $u=0$, p = pressure produced by primer explosion and v = specific volume due to air and primer gases.

Boundary conditions: At breech, $u=0$ for all x. At projectile base, $u_b = 0$ if $p_b < p_{\min}$. Otherwise, $du_b/dt = A(p_b - p_a)/m$.

The partial differential equations can be solved by using an appropriate difference scheme.[1,2] Except for the parameters K_1, K_2, and K_3 all quantities in the model are either specified or can be computed. The three parameters can be estimated from the pressure-time curves, corresponding to a given experimental condition by a nonlinear least-squares method.[2] For the experimental condition in Ref. 1 the present model gives a good fit to the pressure-time curves and predicts 5% unburnt propellant, 7% higher muzzle velocity, and 800°F temperature after primer explosion.

Model for the Experimental Region

The interior ballistic model given by Eq. (1), is applicable at a single experimental condition. In this section, the model is extended to be valid within a preselected zone of experimentation.

The Experimental Region: The experimental region was planned as a two-level factorial design in charge weight, particle diameter, and initial temperature of charge. These three characteristics were chosen as design variables because of their ease of manipulation and the intended applications of the model to charge setting and particle size and charge weight optimization. Two levels for each of the three variables were selected. Particle sizes (0.0205 in. and 0.0263 in.) were chosen sufficiently apart to reflect the range of particle sizes produced by the propellant production process. The temperatures (70°F and 125°F) correspond to the standard temperatures for propellant testing. Values of charge weight (18 grams and 19 grams) were selected to prevent excessively high ignition delay or chamber pressure. Three rounds were fired at each of the 8 experimental points. One design point was replicated to obtain an independent measure of the experimental error. The entire set of 27 experiments was performed in a completely randomized order to eliminate systematic errors. For each round, the experimental pressure-time curve, peak pressure, muzzle velocity, and action time (time between primer explosion and projectile exit at muzzle) were recorded.[2]

Analysis of the Model Parameters: To determine the functional relationship, necessary to build a model valid within the experimental region, K_1, K_2, and K_3 were estimated at the 8 points of the experimental region. The results are given in Table 1. Table 2 contains the analysis of the factorial design with the extimated parameters as responses. Since the effects of X_1, X_2, X_3 on $\hat{K}_2$ and $\hat{K}_3$ are insignificant, K_2 and K_3 do not depend on the experimental conditions and are replaced by their mean values 0.2223 and 0.1692, respectively. K_1 is then re-estimated. These values are given in the last column of Table 1. The last column of Table 2 shows the analysis of the factorial design with $\hat{K}_1$ as response. The effect of X_2 on $\hat{K}_1$ is significant and negative. The effect of X_3 on $\hat{K}_1$ is positive and insignificant but is retained because of its physical significance. The following functional relationship is found to be adequate:

$$\hat{K}_1 \times 10^4 = 2.4225 - 0.1191(X_2 - 0.0234)/0.0029 + 0.0290(X_3 - 97.5)/27.5 \quad (2a)$$

$$\hat{K}_2 = 0.2223; \quad \hat{K}_3 = 0.1682 \quad (2b)$$

A study of the burning characteristics of the propellant provides the physical reasons for the functional relationship given by Eq. (2). Increase in the initial temperature implies larger initial surface area due to particle expansion. Higher initial temperature also leads to a higher final temperature of the products of reation[4] with a consequent increase in the mass burning rate. Since both of these effects are not explicitly considered, K_1 increases with initial temperature. The increase in K_1 with a reduction in particle diameter is due to the neglect of higher heat transfer into the particles for smaller particle size. No physical reasons were found to expect changes in K_2 and K_3 with changes in X_1, X_2, and X_3.

The interior ballistic model applicable within the experimental region is given by Eq. (1) with the burning rate parameters defined by Eq. (2). The predicted pressure-time curves are found to be in close agreement with the observed curves for all the experimental conditions.[2] The predicted peak pressure is 5% larger, the predicted muzzle velocity is 7% larger, and the predicted action time is 10% smaller than the observed values. The predicted unburnt propellant is 3%. These prediction errors are considered to be satisfactory from a practical viewpoint. The errors may be further reduced by empirical corrections.

Table 1 $\hat{K}_1$, $\hat{K}_2$, $\hat{K}_3$ **for the 8 experimental conditions**

X_1	X_2	X_3	$\hat{K}_1 \times 10^4$	$\hat{K}_2$	$\hat{K}_3$	$\hat{K}_1 \times 10^4$
18	0.0205	70	2.6	0.16	0.25	2.5
19	0.0205	70	2.5	0.24	0.19	2.5
18	0.0263	70[a]	2.3	0.24	0.13	2.3
19	0.0263	70	2.3	0.24	0.16	2.3
18	0.0205	125	2.6	0.19	0.20	2.6
19	0.0205	125	2.6	0.23	0.15	2.6
18	0.0263	125	2.3	0.24	0.07	2.3
19	0.0263	125	2.3	0.27	0.14	2.3
18	0.0263	70[a]	2.3	0.20	0.22	2.2

[a] Replicate.

Table 2 Analysis of the factorial design with $\hat{K}_1$, $\hat{K}_2$ **and** $\hat{K}_3$ **as responses**

Variable	Effect on $\hat{K}_1 \times 10^4$	Effect on $\hat{K}_2$	Effect on $\hat{K}_3$	Effect on $\hat{K}_1 \times 10^4$
X_1	−0.009	0.021	−0.008	0.01
X_2	−0.142[a]	0.019	−0.030	−0.119[a]
X_3	0.014	0.008	−0.024	0.029
X_1X_2	0.030[a]	−0.008	0.022	0.008
X_1X_3	0.023[a]	−0.005	0.012	0.014
X_2X_3	−0.008	0.004	−0.003	−0.009
$X_1X_2X_3$	−0.013	0.006	0.009	−0.010
Mean	2.43[a]	0.222[a]	0.168[a]	2.423[a]

[a] Effect is significant at 5% level.

Model Validation

It should be noted that if the relationship between the design variables and the parameters can be assumed to be linear, then the proven adequacy of Eq. (2) implies the validity of the interior ballistic model. As a further check, three confirmation experiments were conducted with propellant consisting of a mixture of the two particle sizes in different proportions (1:3, 1:1, 3:1). For the computation of the mass burning rate, the two particle sizes were considered separately with appropriate K_1 obtained from Eq. (2). All the prediction errors were found to be of the same magnitude as those reported earlier, confirming the validity of the model.

References

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[4] Corner, J., *Theory of Interior Ballistics of Guns*, Wiley, New York, 1950.